## NPS-53-86-010

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NAVAL POSTGRADUATE SCHOOL MONTEREY CALIFORNIA 93943

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20. ABSTRACT (Conflnue on reverse elde If neceeeary and ldentlty by block mumber)

We analyze here the process of coalition formation in the Catalonian Parliament. Using Owen's modification of the Shapley Value (for games with a Priori Unions), we show that the actual result is not unreasonable, given the underlying political structure.
I. Description of situation. Parties involved.

We consider here the coalitional situation inside the Catalonian Parliament after the 1980 elections. Essentially, the Parliament has 135 members; voting is by a straight majority rule, so that any coalition of 68 or more members will win.

In the 1980 elections six parties elected members to Parliament:

| Party | Members |
| :--- | :---: |
| CU (Convergencia i Unio') | 43 |
| PS (Partit dels Socialistes de | 33 |
| PC (Partit Socialista Unificat | 23 |
| de Catalunya) | 18 |
| CR (Centristes de Catalunya) | 14 |
| PA (Partido Socialista de Andalucia) | 2 |

Apart from this, two independents, Benet and Portabella, were elected, giving a total of 135 . We give below a brief ideological description of the parties.

Convergencia i Unio is a nationalist middle-of-the-road coalition, while Esquerra Republicana de Catalunya is a moderately left-of-center nationalist party.

Centristes de Catalunya was a middle-of-the-road party associated with the Union de Centro Democrático, the party which was, at that time, governing Spain.

The Partit dels Socialistes de Catalunya, a left-of-center party, is associated with the Partido Socialista Obrero Español, which was at that time the main opposition party at the national level.

The Partit Socialista Unificat de Catalunya is a communist party, with similar policies to the Partido Comunista de España.

The Partido Socialista de Andalucía is a radical left-wing party which tried to represent the Andalusian immigrants in Caralonia.

Finally, the two independent members were closely allied to the communist party.
II. Measures of Power.

We will use the Shapley value |Shapley, 1953| as a measure of power in abstract situations. Where, however, the ideological situation suggests that some coalitions are more likely than others, we will use Owen's modification |1977| to games with a priori unions.

We refer the reader to the articles cited for mathematical details of these values. (See also Hart and Kurz, 1984.)

The formal structure of the Parliament is an 8-element set

$$
N=\{C U, P S, P C, C C, E R, P A, b, p\}
$$

with votes as given above ( $b$ and $p$ are the two independents, with one vote each).

We then have $v(S)=1$ for any coalition $S$ with 68 or more votes; $v(S)=0$ for other coalitions.

It is not difficult to calculate the Shapley value for this game, which is given by the second column in Table 1 below.

Note that CC and ER, with $13.33 \%$ and $10.37 \%$ respectively of the seats, have considerably less power - $9.29 \%$ in each case. On the other hand PA with $1.48 \%$ of the seats has $2.62 \%$ of the power.
III. The Coalition Structures.

We proceed now to modify the value by inserting coalitional considerations.

The first modification is due to the fact that both independent members (b and p) were closely allied to the communist party - were in fact included in its lists. This gives rise to the coalition structure

$$
\{C U\},\{P S\},\{P C, b, p\},\{C C\},\{E R\},\{P A\} .
$$

With this structure, the modified value now is given by the third column in Table 1.

- Table 1

| Party | Votes | Value | $\frac{\text { Modified }}{\text { value }}$ |
| :--- | :---: | :---: | :---: |
| CU | .3185 | .3405 | .4000 |
| PS | .2444 | .2595 | .2333 |
| PC | .1704 | .1738 | .1889 |
| CC | .1333 | .0929 | .0667 |
| ER | .1037 | .0929 | .0667 |
| PA | .0148 | .0262 | 0 |
| b | .0074 | .0071 | .0222 |
| P | .0074 | .0071 | .0222 |

We note that PA has, in effect, become a dummy. It is possible, as we see, to distinguish the independents from the rest of the PC, but the effect is small enough that we will disregard it and simply treat \{PC,b,p\} as a single block. We have then a game with five players, CU, PS, PC, CC, ER, plus a dummy, PA. The power is given in the first column of Table 2 below.

Now the coalitions become more interesting. On the one hand, a coalition between the two left-wing parties, PS and PC, seems natural (in fact, PC would seem to have nowhere else to go). This would give us a structure

$$
\{C U\},\{P S, P C\},\{C C\},\{E R\},
$$

with the value given in the second column of Table 2.

On the other hand, $C U$ also had a possible coalition partner in CC. In fact, CU had supported the Union de Centro Democratico in Spain's national government. It was only normal for the coalition \{CU,CC\} to form in Catalonia. This would give, instead, the structure

$$
\{C U, C C\},\{P S\},\{P C\},\{E R\},
$$

with value given in Table 2, third column.

Of course, both protocoalitions, \{CU,CC\} and \{PS,PC\}, could form, giving the structure

$$
\{C U, C C\},\{P S, P C\},\{E R\},
$$

with value given in Table 2, fourth column.

At this point, of course, the moderately left party, ER, found itself in an enviable position. With only 14 members (just over $10 \%$ ), it now held the balance of power: it could, in effect, determine whether Catalonia would have a center-right or left-of-center government. In either case, the opposite coalition would probably dissolve. We have, then, two alternatives. If ER joins the center-right coalition, we would have the structure $\{\{C U, C C\},\{P S\},\{P C\}$
with value in Table 2, fifth column, whereas, if ER joins the left-wing coalition, we have

$$
\{\{C U\},\{C C\},\{\{P S, P C\}, E R\}
$$

with value in Table 2, sixth column.

In either case, $E R$ would seem to be equally well, and it would be a question only as to ER's preferences. In fact, the first case occurred: a right-wing coalition $\{\{C U, C C\}, E R\}$ formed, in which $C U$ had the bulk of the power. Strangely enough (though not so strangely given our analysis), ER with 14 seats has greater power than CC with 18 seats, and this is reflected in the actual division of power since ER had greater power (in terms of cabinet consellers) than CC in the actual government formed. But of course this is due to CC's "debt" to CU for CU's help in the national government.

## Table 2

| Party | $\frac{\text { initial }}{\text { game }}$ | \{PS, PC \} | $\underline{\text { \{CU,CC \} }}$ | $\begin{aligned} & \frac{\{P S, P C\}}{\text { and }} \\ & \{C U, C C\} \end{aligned}$ | \{ \{CU, CC \}, ER \} | $\underline{\{ } \underline{\text { PS }}, \mathrm{PC}\}, \mathrm{ER}\}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CU | . 4000 | . 1667 | . 4167 | . 1667 | . 5000 | 0 |
| PS | . 2333 | . 2500 | . 1667 | . 1667 | 0 | . 3333 |
| PC | . 2333 | . 2500 | . 1667 | . 1667 | 0 | . 3333 |
| CC | . 0667 | . 1667 | . 0833 | . 1667 | . 1667 | 0 |
| ER | . 0667 | . 1667 | . 1667 | . 3333 | . 3333 | . 3333 |

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